
**Liquid Crystals**
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

# Optical transmission of chevron-type ferroelectric liquid crystal displays

Alexis De Vos[a]
[a] Laboratorium voor Elektronika en Meettechniek, Rijksuniversiteit te Gent, Gent, Belgium

**To cite this Article** De Vos, Alexis(1989) 'Optical transmission of chevron-type ferroelectric liquid crystal displays', Liquid Crystals, 6: 3, 373 — 381
**To link to this Article: DOI:** 10.1080/02678298908029088
**URL:** http://dx.doi.org/10.1080/02678298908029088

LIQUID CRYSTALS, 1989, VOL. 6, NO. 3, 373–381

# Optical transmission of chevron-type ferroelectric liquid crystal displays

by ALEXIS DE VOS
Laboratorium voor Elektronika en Meettechniek, Rijksuniversiteit te Gent, Sint Pietersnieuwstraat 41, B-9000 Gent, Belgium

(*Received 2 February 1989; accepted 23 April 1989*)

Recent X-ray diffraction studies have revealed that the smectic C dielectric in ferroelectric liquid crystal displays quite generally has a *chevron* structure, instead of the simple *bookshelf* geometry. The influence of both the tilt of the smectic C layers and the discontinuity of the latter tilt are investigated here. An analytical expression is deduced for the transmission of normally incident light.



## 1. Introduction

Rieker *et al.* [1] and Ouchi *et al.* [2] have recently demonstrated by means of X-ray diffraction that surface-stabilized ferroelectric liquid crystals quite generally have a so-called *chevron* geometry. This means that the smectic C layers are not planes, but display a sharp break in a plane parallel to the two surfaces. Figure 1 shows the various models that have been used to describe a ferroelectric liquid crystal display. Model (*a*) is the classical *bookshelf* geometry, where the smectic layers are both flat and perpendicular to the substrates. In model (*b*) the layers are still flat, but tilted over an angle $\delta$. Model (*c*) is the symmetric *chevron* structure, where the angle $\delta$ changes sign abruptly in the mid-plane, i.e. the plane half-way between the two substrates. Finally, model (*d*) is the most general model accepted nowadays, namely the asymmetric *chevron* geometry, where the layer tilt changes sign in a plane parallel to the substrates, but not located half-way between them. Which of the four models is actually applicable to a particular liquid crystal device depends not only upon the actual liquid crystal mixture, but also upon the actual alignment material [2], the thermal history of the cell [1] and even the electrical history of the cell [3].

Here we calculate the transmission of light incident perpendicularly on a liquid crystal device. The *bookshelf* geometry has already been treated by De Vos and Reynaerts [4]. In §2 their model is generalized to the tilted-layer model and in §3 to the *chevron* structure.

## 2. Tilted-layer devices

In this section we take the liquid crystal layer to be completely uniform; that is, both the layer tilt $\delta$ and the director azimuthal angle $\varphi$ are independent of the positional coordinates $x$, $y$ and $z$. Figure 2 shows the geometrical properties of the material. The smectic directors are constrained to stay on the surface of a cone with a cone angle $2\vartheta$ and with an axis making the angle $\delta$ with the $(x, y)$ plane, i.e. the plane parallel to the two substrates. In the literature $\vartheta$ is called the tilt angle, however, it could be named the director tilt, in order to distinguish it from the layer tilt $\delta$. In the $(x, y)$ plane the direction of the $x$ axis is chosen along the rubbing direction of the

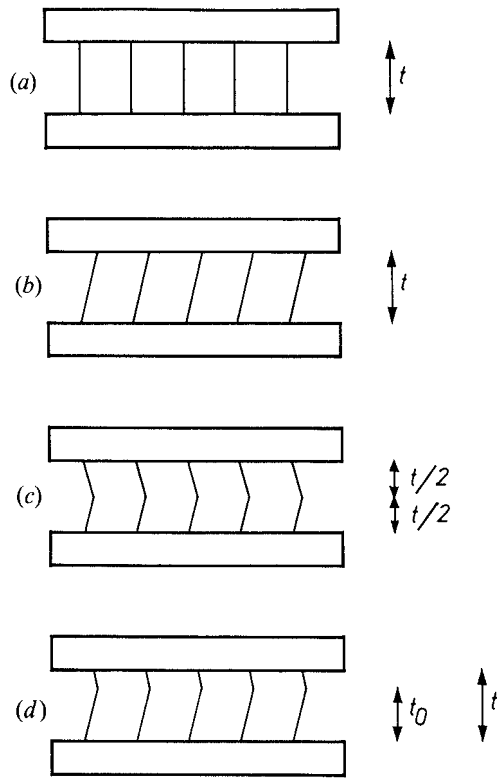


Figure 1. Different models for surface-stabilized ferroelectric liquid-crystal displays: (*a*) *bookshelf* geometry; (*b*) tilted layer geometry; (*c*) symmetric *chevron* geometry; (*d*) asymmetric *chevron* geometry.

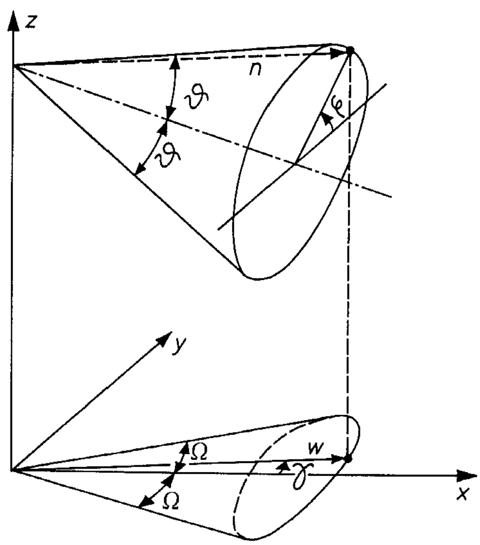


Figure 2. The smectic C cone and its projection onto the ($x$, $y$) plane, i.e. the plane parallel to the stabilizing surfaces.

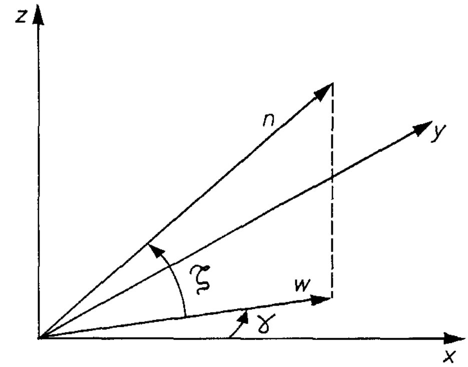


Figure 3. Definition of the Euler angles $\zeta$ and $\gamma$ for the director.

alignment layers. The azimuthal angle $\varphi$ of the director can vary freely under the influence of viscoelastic and electric forces. The origin of the angle $\varphi$ (i.e. the reference point $\varphi = 0$) is chosen such that the corresponding $c$ director is parallel to the $y$ axis. The projection of the smectic cone on the $(x, y)$ plane displays a cone angle $2\Omega$, given by

$$\tan \Omega = \left(1 - \frac{\sin^2 \delta}{\cos^2 \vartheta}\right)^{-1/2} \tan \vartheta$$

The projection of the director $\mathbf{n}$ onto the $(x, y)$ plane is denoted by $w$ and makes an angle $\zeta$ with $\mathbf{n}$ and an angle $\gamma$ with $x$; see figure 3. We have

$$\sin \zeta = \cos \delta \sin \vartheta \sin \varphi - \sin \delta \cos \vartheta, \quad (1)$$

$$\tan \gamma = \frac{\sin \vartheta \cos \varphi}{\cos \delta \cos \vartheta + \sin \delta \sin \vartheta \sin \varphi}. \quad (2)$$

We now consider a light beam incident perpendicularly on the display, i.e. propagating along the $z$ axis. The free-space wavenumber is $k$. A polarizer has polarized the incoming light such that the electric vector at $z = 0$ makes an angle $\gamma_p$ with the $x$ axis; see figure 4. The electric vector of the light wave can be decomposed into an extraordinary component parallel to $w$ and an ordinary component parallel to $u$, the normal to $w$ in the $(x, y)$ plane; see figure 4. The ordinary wave has an electric vector

$$\mathbf{E}_o = \sin (\gamma_p - \gamma) \exp (ikn_o z)\mathbf{1}_u,$$

whereas the extraordinary wave has an electric vector

$$\mathbf{E}_e = \cos (\gamma_p - \gamma) \exp (ikn_e z)\mathbf{1}_w.$$

In these equations $n_o$ and $n_e$ are the refractive indices associated with the ordinary and extraordinary waves respectively; thus

$$n_o = \varepsilon_\perp^{1/2}, \quad (3)$$

$$n_e = \left(\frac{\varepsilon_\parallel \varepsilon_\perp}{\varepsilon_\parallel \sin^2 \zeta + \varepsilon_\perp \cos^2 \zeta}\right)^{1/2}, \quad (4)$$

where $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the dielectric constants (at optical frequencies) of the liquid crystal, respectively perpendicular to and along the director.

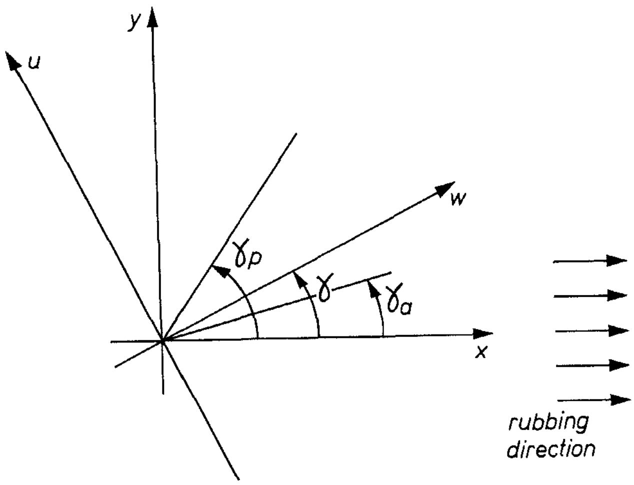


Figure 4. The $(x, y)$ plane, i.e. the plane parallel to the stabilizing surfaces, with the orientation of polarizer and analyser.



At $z = t$ both waves leave the liquid crystal layer. We suppose that the analyser makes an angle $\gamma_a$ with the $x$ axis. The components of the ordinary and extraordinary electric vector along the transmitting axis of the analyser are

$$-\sin(\gamma - \gamma_a)\sin(\gamma_p - \gamma)\exp(ikn_o t)$$

and

$$\cos(\gamma - \gamma_a)\cos(\gamma_p - \gamma)\exp(ikn_e t)$$

respectively.

After summing and taking the magnitude, we obtain the square root of the light intensity transmission as

$$\tau^{1/2} = |\cos(\gamma - \gamma_a)\cos(\gamma_p - \gamma)\exp(ikn_e t) - \sin(\gamma - \gamma_a)\sin(\gamma_p - \gamma)\exp(ikn_o t)|.$$

As a function of $kt$, this magnitude shows two kinds of extremum:

(i) $\cos(\gamma - \gamma_a)\cos(\gamma_p - \gamma) - \sin(\gamma - \gamma_a)\sin(\gamma_p - \gamma)$,
(ii) $\cos(\gamma - \gamma_a)\cos(\gamma_p - \gamma) + \sin(\gamma - \gamma_a)\sin(\gamma_p - \gamma)$,

which can both be simplified to give

(i) $\cos(\gamma_p - \gamma_a)$,
(ii) $\cos(2\gamma - \gamma_p - \gamma_a)$.

The first kind of extremum, i.e.

$$\tau = \cos^2(\gamma_p - \gamma_a),$$

is independent of the director orientation and therefore useless for display purposes. The second kind of extremum, i.e.

$$\tau = \cos^2(2\gamma - \gamma_p - \gamma_a),$$

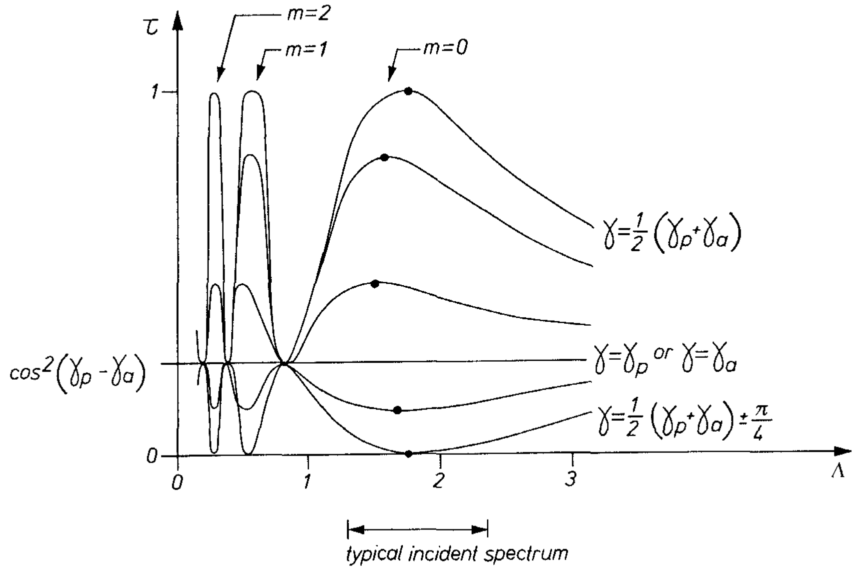


Figure 5. Optical transmission $\tau$ as a function of the dimensionless wavelength $\Lambda = \lambda/(t\,\Delta n)$ and the Euler angle $\gamma$.



depends upon $\gamma$ and thus on the director orientation, and can therefore be employed for display purposes. Such an extremum is realized under the condition

$$k(n_o - n_e)t = (2m + 1)\pi, \tag{5}$$

with $m$ an arbitrary integer (positive or zero). With equation (1), we obtain

$$\begin{aligned} n_o - n_e &= \varepsilon_\perp^{1/2} - \left[\frac{\varepsilon_\parallel \varepsilon_\perp}{\varepsilon_\perp + (\varepsilon_\parallel - \varepsilon_\perp)(\cos\delta \sin\vartheta \sin\varphi - \sin\delta\cos\vartheta)^2}\right]^{1/2}, \\ &\approx \Delta n\,[1 - (\cos\delta\sin\vartheta\sin\varphi - \sin\delta\cos\vartheta)^2], \end{aligned}$$

where $\Delta n = \varepsilon_\perp^{1/2} - \varepsilon_\parallel^{1/2}$ is the optical anisotropy of the liquid crystal. Thus for the first-order birefringence, i.e. for $m = 0$, equation (5) becomes

$$\frac{2t}{\lambda}\,\Delta n\,[1 - (\cos\delta\sin\vartheta\sin\varphi - \sin\delta\cos\vartheta)^2] = 1$$

or

$$\frac{\lambda}{t\,\Delta n} = 2[1 - (\cos\delta\sin\vartheta\sin\varphi - \sin\delta\cos\vartheta)^2].$$

Figure 5 shows the transmission $\tau$ as a function of the dimensionless wavelength $\Lambda = \lambda/(t\,\Delta n)$. We see that the value of $\Lambda$ for which $\tau$ equals its ($m = 0$) extremum shifts little on varying the angle $\gamma$, i.e. during switching. Indeed it is possible to demonstrate that the extreme value of $\tau$ occurs for a variable $\Lambda$, which is constrained by $2\cos^2\vartheta \leqslant \Lambda \leqslant 2$. If we suppose that the incident light wave has a spectrum well matched with the broad extremum of figure 5 then the light is transmitted with approximately the extreme value of $\tau$, irrespective of the instantaneous value of $\gamma$.

We recall that this extreme value is given by

$$\tau \approx \tau_{extr} = \cos^2(2\gamma - \gamma_p - \gamma_a). \tag{6}$$

We now introduce the two stable states of the device. They are characterized by the azimuthal angles $\varphi = \varphi_0$ (the up state) and $\varphi = \pi - \varphi_0$ (the down state) respectively. The value of $\varphi_0$ (often called the *pretilt*) will be discussed later (see §§4 and 5). Corresponding to the two stable $\varphi$ values, we have two stable $\gamma$ values, namely $\gamma_0$ and $-\gamma_0$ respectively. In accord with equation (2), we have, of course,

$$\tan\gamma_0 = \frac{\sin\vartheta\cos\varphi_0}{\cos\delta\cos\vartheta + \sin\delta\sin\vartheta\sin\varphi_0}.$$

For maximum contrast of the liquid crystal device, we wish $\tau$ to be zero either for $\gamma = \gamma_0$ (up state) or for $\gamma = -\gamma_0$ (down state). If we make the choice up = black and down = white, equation (6) then requires

$$\cos(2\gamma_0 - \gamma_p - \gamma_a) = 0,$$

i.e. we have to orient the two polarizers such that

$$\gamma_a + \gamma_p = 2\gamma_0 - \tfrac{1}{2}\pi, \tag{7}$$

the most frequently applied (but not unique) method being $\gamma_a = \gamma_0$ and $\gamma_p = \gamma_0 - \frac{1}{2}\pi$ (crossed polarizers). Substituting equation (7) into equation (6) yields

$$\tau \approx \sin^2(2\gamma_0 - 2\gamma).$$

This equation expresses $\tau$ as a function of the director orientation in terms of the Euler angle $\gamma$. The value of $\tau$ varies from 0 for $\gamma = \gamma_0$ (up state) through $\sin^2 2\gamma_0$ for $\gamma = 0$ to $\sin^2 4\gamma_0$ for $\gamma = -\gamma_0$ (down state). If we want to express the director orientation in terms of the azimuthal angle $\varphi$, we first transform the equation by straightforward trigonometry to

$$\tau \approx \sin^2 2\gamma_0 \frac{(1 - \cot\gamma_0\tan\gamma)^2(1 + \tan\gamma_0\tan\gamma)^2}{(1 + \tan^2\gamma)^2}$$

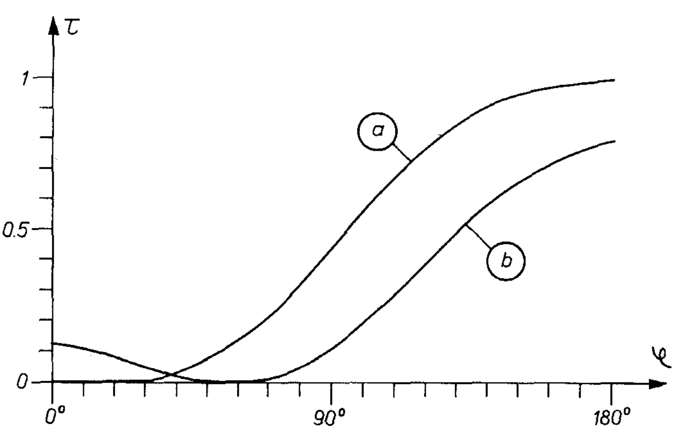


Figure 6. Optical transmission $\tau$ as a function of director azimuthal angle $\varphi$. The smectic-layer tilt $\delta$ and the smectic cone angle $\vartheta$ are chosen as 17° and 20° respectively. (*a*) Polarizers oriented such that $\varphi = 0°$ corresponds to black; (*b*) polarizers oriented such that $\varphi = \text{Arcsin}(\tan\delta\cot\vartheta) = 57°$ corresponds to black.

and then apply equation (2) in order to find

$$\tau \approx \frac{4 \tan^2 \Theta \, c^2(\varphi_0)}{[1 + \tan^2 \Theta \, c^2(\varphi_0)]^2} \frac{\left[1 - \dfrac{c(\varphi)}{c(\varphi_0)}\right]^2 [1 + \tan^2 \Theta \, c(\varphi_0) c(\varphi)]^2}{[1 + \tan^2 \Theta \, c^2(\varphi)]^2}, \tag{8}$$

where $c(x)$ is a cosine-like function, defined by

$$c(x) = \frac{\cos x}{1 + \tan \delta \tan \vartheta \sin x}$$

and where the angle $\Theta$ is given by

$$\tan \Theta = \frac{\tan \vartheta}{\cos \delta}.$$

Figures 6 shows some examples of curves $\tau(\varphi)$. We have chosen $\vartheta = 20°$ and $\delta = 17°$ in order to fit the results of Hartmann [3] for the liquid-crystal mixture ZLI-3234.

## 3. *Chevron* devices

If the liquid crystal has a *chevron* geometry, the smectic layer tilt equals $\delta$ for $0 < z < t_0$ and equals $-\delta$ for $t_0 < z < t$. At $z = t_0$ we suppose a discontinuity, i.e. an infinitely thin domain wall. Since the stabilizing surfaces favour director orientations as horizontal as possible, switching from the up state to the down state does not happen in an arbitrary sense, as it does in the *bookshelf* geometry. In the lower cell

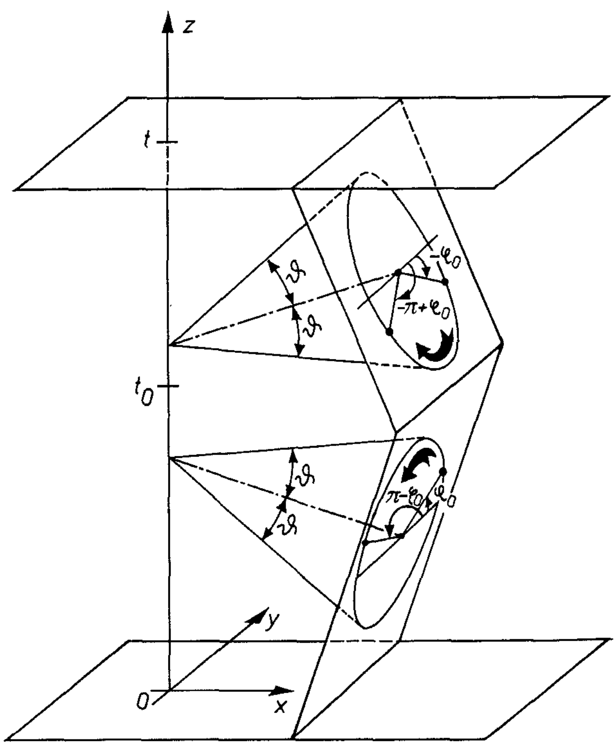


Figure 7. Switching in chevron-type liquid crystal cells.

positive values of $\varphi$ are favoured and the director rotates clockwise ($\varphi_0 \leqslant \varphi \leqslant \pi - \varphi_0$), while in the upper cell the director rotates counterclockwise ($-\varphi_0 \geqslant \varphi \geqslant -\pi + \varphi_0$); see figure 7. The fact that the directors rotate in different senses in the different parts of the liquid crystal was already mentioned by Xue *et al.* [5]. Thus, at any moment, the directors in different cell halves have opposite azimuthal angles $\varphi$.

Now, opposite values of both $\delta$ and $\varphi$ yield opposite values of the angle $\zeta$ and equal values of the angle $\gamma$; see equations (1) and (2). The opposite values of $\zeta$ give rise to equal values of the refractive indices $n_o$ and $n_e$ (see equations (3) and (4)), whereas the equal values of $\gamma$ result in equal decomposition of the electric vector into ordinary and extraordinary components. These two facts together result in an absence of any optical discontinuity. In spite of the material discontinuity at $z = t_0$, the light experiences no reflections at the interface. We can conclude that the *chevron* geometry, from an optical point of view, behaves completely like the tilted layer geometry. All of the results of §2 are therefore applicable to liquid crystals with a *chevron* structure, either with a symmetric or an asymmetric *chevron* geometry.

## 4. The case $\varphi_0 = 0$

If in equilibrium a high enough electric field $E$ is applied, the stable states are characterized by the absence of any torque $EP \cos \delta \sin \varphi$, which would make the liquid-crystal director rotate on its cone. Therefore the two stable states are $\varphi = 0$ and $\varphi = \pi$. In other words, $\varphi_0 = 0$. We immediately have $\gamma_0 = \Theta$, and formula (8) becomes

$$\tau \approx \sin^2 2\Theta \frac{\left[1 - \dfrac{c(\varphi)}{c(\varphi_0)}\right]^2 [1 + \tan^2 \Theta\, c(\varphi_0)c(\varphi)]^2}{[1 + \tan^2 \Theta\, c^2(\varphi)]^2}. \tag{9}$$

This function $\tau(\varphi)$ varies from $\tau = 0$ at $\varphi = 0$ (up state) through $\tau = \sin^2 2\Theta$ at $\varphi = \frac{1}{2}\pi$ to $\tau = \sin^2 4\Theta$ at $\varphi = \pi$ (down state). Figure 6 (*a*) is an example where $\gamma_0 = \Theta$ equals 21°. The best display action is obtained if $\tau$ equals unity in the down state, and this happens if $\sin^2 4\Theta = 1$, or $\Theta = \frac{1}{8}\pi$.

## 5. The case $\varphi_0 = \text{Arcsin}(\tan \delta \cot \vartheta)$

If the liquid crystal cell is operated such that at equilibrium no electric field is present then the stable states are governed by the surface stabilization that orients the liquid crystal directors parallel to the substrates. Therefore the two stable states are given by $\zeta = 0$ or $\varphi = \text{Arcsin}(\tan \delta \cot \vartheta)$ and $\varphi = \pi - \text{Arcsin}(\tan \delta \cot \vartheta)$. The angle $\varphi_0 = \text{Arcsin}(\tan \delta \cot \vartheta)$ was introduced by Hartmann [3], Xue *et al.* [5] and Chieu *et al.* [6]. The expression for $\varphi_0$ implicitely assumes that $\delta < \vartheta$. If $\delta$ were larger than $\vartheta$ then no horizontal generatrix would exist on the smectic cone, and surface stabilization would not be bistable, since the surface would favour only one orientation ($\varphi = \frac{1}{2}\pi$).

After some calculations, we find $\gamma_0 = \Xi$, and equation (8) becomes

$$\tau \approx \sin^2 2\Xi \frac{\left[1 - \dfrac{c(\varphi)}{c(\varphi_0)}\right]^2 [1 + \tan^2 \Theta\, c(\varphi_0)c(\varphi)]^2}{[1 + \tan^2 \Theta\, c^2(\varphi)]^2}, \tag{10}$$

where the angle $\Xi$ is given by

$$\tan \Xi = \tan \vartheta \left(1 - \frac{\sin^2 \delta}{\sin^2 \vartheta}\right)^{1/2}.$$

This function $\tau(\varphi)$ varies from $\tau = 0$ at $\varphi = \varphi_0$ (up state) through $\tau = \sin^2 2\Xi$ at $\varphi = \frac{1}{2}\pi$ to $\tau = \sin^2 4\Xi$ at $\varphi = \pi - \varphi_0$ (down state). Figure 6 (*b*) gives an example where $\gamma_0 = \Xi$ equals 10°. Optimum display performance is now obtained for $\sin^2 4\Xi = 1$, or $\Xi = \frac{1}{8}\pi$.

## 6. The *bookshelf* geometry

In the special case $\delta = 0$, i.e. the *bookshelf* geometry, $\varphi_0$ equals 0 in both the high-field and low-field cases. Indeed, both the electric forces and the boundary forces tend to orient the directors horizontally. For $\delta = 0$ all three angles $\Omega$, $\Theta$ and $\Xi$ equal the half-cone angle $\vartheta$. Further application of $\delta = 0$ to either equation (9) or equation (10) yields

$$\tau \approx \sin^2 2\vartheta \frac{(1 - \cos \varphi)^2 (1 + \tan^2 \vartheta \cos \varphi)^2}{(1 + \tan^2 \vartheta \cos^2 \varphi)^2}, \tag{11}$$

i.e. the formula published by De Vos and Reynaerts [4]. Optimum display performance is obtained for $\vartheta = \frac{1}{8}\pi$, a fact that was recognized by Lagerwall *et al.* [7].

## 7. Conclusions

We have described a surface-stabilized ferroelectric liquid crystal cell as a stack of two layers. We have assumed that each of these is a uniform smectic. Both smectic layers have the same half-cone angle $\vartheta$, but opposite layer tilt $\delta$ and opposite director azimuthal angle $\varphi$. Under these conditions, normally incident light undergoes no reflections at the interface between the two cell parts. The optical transmission $\tau$ of such *chevron* liquid crystal cells can be calculated analytically. The result is given by equation (8), expressing $\tau$ as a function of the instantaneous director azimuthal angle $\varphi$. The formula is a generalization of equation (11), which has been published before [4] but is only valid for *bookshelf* liquid-crystal cells ($\delta = 0$).

The author is a research assistant in the framework of the concerted research project 'II–VI semiconductors' of the Belgian State (Science Policy Programming Services).

## References

[1] Rieker, T., Clark, N., Smith, G., Parmar, D., Sirota, E., and Safinya, C., 1987, *Phys. Rev. Lett.*, **59**, 2658.
[2] Ouchi, Y., Lee, J., Takezoe, H., Fukuda, A., Kondo, K., Kitamura, T., and Mukoh, A., 1988, *Japan. J. appl. Phys.*, **27**, L725.
[3] Hartmann, W., 1988, *Ferroelectrics*, **85**, 67.
[4] De Vos, A., and Reynaerts, C., 1989, *J. appl. Phys.*, **65**, 2616.
[5] Xue, J., Handschy, M. A., and Clark, N. A., 1987, *Liq. Crystals*, **2**, 707.
[6] Chieu, T., Yang, K., and Lien, A., 1988, *Proceedings of the 1988 International Display Research Conference*, San Diego, edited by L. Weber (I.E.E.E.), p. 178.
[7] Lagerwall, S., Wahl, J., and Clark, N. A., 1985, *Proceedings of the 1985 International Display Research Conference*, San Diego, edited by A. Penz (I.E.E.E.), p. 213.